The Hong Kong University of Science and Technology

Department of Mathematics

PhD THESIS EXAMINATION

Mathematical Theory of Computational Resolution Limit and Efficient Fast Algorithms for Super-resolution

By

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ABSTRACT

The ultimate resolution limit of an imaging system is a long-standing puzzle that originated from the 18th century. Although many heuristic resolution criteria have been proposed in the past century and a half, the exact characterization of the resolution limit is still unknown. In this thesis, we aim to provide a rigorous mathematical theory for the resolution limits of various super-resolution problems. Specifically, we define and quantitatively estimate the computational resolution limits for number detection and stable support recovery in the deconvolution problem, the one and multi-dimensional line spectral estimation problem, and the super-resolution from multi-illumination imaging. The results explicitly elucidate the dependence of each computational resolution limit on the cutoff frequency, signal-to-noise ratio, sparsity of the source, and the illumination pattern. In contrast to the classical resolution limits that only related to the cutoff frequency, our results highlight the importance of all the aforementioned factors to the super-resolution problem. The new understanding and strict characterizations are more important for the super-resolution of modern imaging techniques. Our results also demonstrate the phase transitions in these reconstruction problems. We derive many number detection and support recovery algorithms and numerically confirm the predicted phase transition phenomenons. Finally, we propose a fast algorithm for super-resolving point sources with cluster structure.

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Time: 3:00 p.m.

Venue: Online via Zoom

https://hkust.zoom.us/j/92432022890 (Passcode: hkust)

Thesis Examination Committee:

Chairman	:	**Prof Iam Keong SOU, PHYS/HKUST**
Thesis Supervisor	:	**Prof Hai ZHANG, MATH/HKUST**
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(Open to all faculty and students)

The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).